Pricing of Options

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ABSTRACT

This paper takes the initiative to value stock options by using the Nobel winning Black-Scholes-Merton Model and the Binomial Options Pricing Model. The sole objective of this paper is to investigate whether the models can be utilized to generate investment opportunities for Calls and Puts by valuation. Ten of the most well known large cap U.S firms companies have been selected for this paper. The paper begins with a brief introduction of Derivative products followed by the literature review which covers some of the most well known models for the pricing of options. The fourth portion analyses the empirical findings after the paragraph on methodology. The concluding paragraph sheds light on the application aspect of the models for identifying potential options.

Keywords: Derivatives, Option Valuation, Black Scholes Merton, Binomial Model

INTRODUCTION

To define in the simplest term, Derivatives is nothing short of a financial instrument that can be purchased and sold in respective exchanges similar to shares and fixed income securities. There are of course major variations which this paper does not seek to examine. In the past thirty years, the Derivatives markets in the west have grown to over trillion dollars. The U.S derivatives market is reported to be over $600 trillion. Some of the well known exchanges include the Chicago Mercantile Exchange (CME) and New York Stock Exchange Euronext (NYSE Euronext). Derivatives comprises of two central products, more commonly known as "Options and Futures." These two instruments are essentially two umbrellas that provide other financial instruments which can also be traded for hedging and speculation purpose. A Futures contract is an agreement between buyer and seller to exchange a specified financial asset at a specified price at a specified future date. There are several types of future contracts; some of the common include commodities, financials, metals and energy. Other derivative instruments include weather derivatives and swap contracts. An interesting fact on futures trading is the absence of the delivery of the asset even though the asset, price and expiration are specified. Only cash transaction takes place with very little delivery on actual traded goods. Futures trading contains considerable amount of risk, more than equities due its volatility and leverage. Once an investor enters into a futures contract, there is an obligation to dispose the contract; however, in the case of options there is no obligation but rather a choice for the investor to either exercise or sell the underlying contract. Options are divided into two categories, Calls and Puts. A call option gives the right to purchase an underlying financial asset at a specified price within a specified time. A put option gives the holder the right to sell an underlying security at a specified price within a specified time. This study is aimed to test two of the well known option models applicability in seeking investment opportunities. Valuating a financial security is a complex task and derivative instruments are no exception. Valuation is performed to seek underpriced and overpriced securities. Some of the well known option valuation models have been discussed in the following literature section.

LITERATURE

The Black-Scholes Model was developed by Fischer Black and Myron Scholes, two Economists who won the Nobel Prize in 1997 in Economics for their contribution to derivatives. Their paper "The Pricing of Options & Corporate Liabilities" was featured in the Journal of Political Economy introduced the world to the Black-Scholes model. Their model can be used to price options, options on commodities, financial assets as well as employee stock options. The model was developed to value European options (Black & Myron, 1973). The model uses several variables including, the stock and strike price, risk free rate, time and volatility. It also incorporates the exponential function and the standard normal cumulative distribution function. There are several assumptions that underlie the model as a result the model is beneficial to price European options. The model can be applied to price American options that pay no dividend and in most cases, American options are usually exercised towards the end of expiry. The six assumptions of the model include:

(1) No dividends are paid on the underlying stock.
(2) Markets are efficient.
(3) No commissions.
(4) Interest rates are fixed.
(5) The option can be exercised at expiry.
(6) Stock returns follow lognormal distribution.

Price of Call Option = $SN(d_1) - \frac{X}{e^{rt}} N(d_2)$

Price of Put Option = $\frac{X}{e^{rt}} N(-d_2) - SN(-d_1)$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)(t)}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)(t)}{\sigma\sqrt{t}}$$

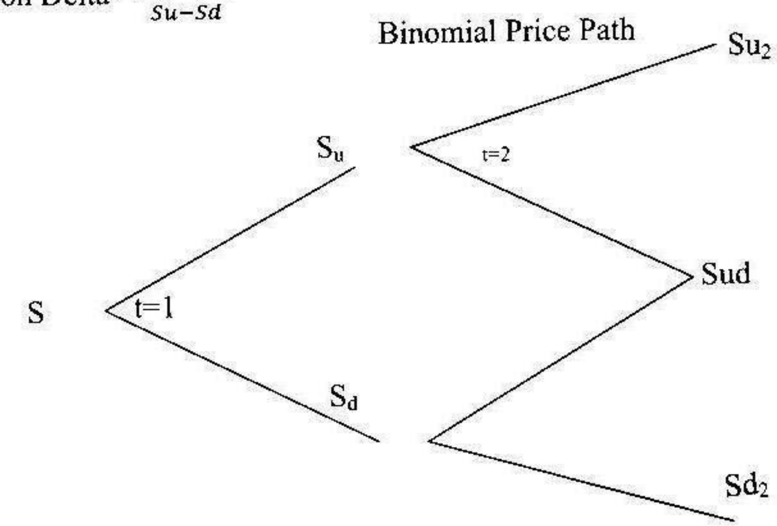
Variable	Description
S	Stock Price
X	Exercise Price
t	Time of option contract
r	Risk free rate
ó	Volatility of stock price
Ln	Natural Logarithm
N(d)	Cumulative distribution function
e	Exponential function

Of the six assumptions, four of them do not reflect the reality of imperfect markets. Commissions do exist in options trading and interest rates are never fixed and differ for both lending and borrowing. The model also utilizes the Treasury-Bill rate and assumes the asset is riskless. Whether markets are efficient and stocks actually do follow lognormal distribution are debatable topics in finance. These assumptions tend to limit the model which may not lead to accurate pricing.

The Binomial Pricing Model (1976)

Value of Call = Current value of underlying asset x Option Delta- Borrowing needed to replicate the option.

Option Delta = $\frac{Cu - Cd}{Su - Sd}$



Variable	Description
Cu	Value of call option if stock price increases to Su.
Cd	Value of call option if stock price decreases to Sd.
Su	Increase in stock price from S.
Sd	Decrease in stock price from S.
S	Initial Stock Price

[redacted]

The Binomial model is based upon the asset price for a specified time period in which the asset price can move either of two directions, an increase or decrease in the price of the asset. This particular model has unique characteristics and can be used to value both European as well as American options (Chung & Shih, 2007). There are several sequences involved when utilizing this model. The first step involves creating the price path. The possible price movements of the asset price have to be estimated to form the tree as displayed above. The model then breaks down the time to expiration into various path or time intervals. As shown above, the Binomial tree is established beginning with the price of the asset at "S" and possible prices for the time intervals. At the end of each tree, the payoff of the option is calculated which is equal to its intrinsic value and works backward for each time tree. The value is then utilized in other trees to arrive at the base node. The model assumes markets are efficient, zero opportunities for arbitrage, reduces the possibility of price changes and risk neutrality. As stated earlier, the concept of efficient market, risk neutrality and zero opportunities for arbitrage are debatable. The model selects certain prices for the binomial price path out of several for each particular node and the justification for the prices is not explored and neither based on a forecasted model of the particular asset which can lead to inaccurate value of the underlying option.

RISK NEUTRAL VALUATION

An alternate methodology using the binomial path is the risk neutral approach which also utilizes the binomial path. This particular model provides the same answer as the above model except it uses the probability variable and can be a calculated in a shorter span (Broadie & Jerome, 2004; Hull, 2008).

$$p = \frac{e^{rt} - d}{u - d}$$

$$Option\ Price = \frac{p^2.VSu_2 + 2p.(1-p).VSud + (1-p)^2.VSd_2}{e^{2rt}}$$

Variable	Description
P	probability
R	Risk free rate
D	1 - (% decrease in stock price)
U	1 + (% increase in stock price)
T	time
VSu_2	intrinsic value of stock in Su2
VSud	intrinsic value of stock in Sud
VSd_2	intrinsic value of stock in Sd2
E	exponential function

The most difficult aspect in utilizing both the methodologies for the Binomial model is estimating the exact movement in stock price. In both approaches the movement in stock prices is estimated. A major limitation to this model is the price movement and creating the binomial tree which is based on assumptions. In most valuations, fewer binomial trees will result in mixed findings. The larger the binomial path accompanied with accurate estimated future prices can provide accurate option prices.

Black-Scholes Dividend Paying Model (1979)

This model is an extension of the Black-Scholes model and can be used to price dividend paying European as well as American options. The assumptions and limitations are identical to that of the Black-Scholes. More commonly known as the Black-Scholes-Merton model, Robert C. Merton modified to include the dividend yield for pricing European options (Hull, 2008). This particular equation can also be applied to value American options. The Black-Scholes model was limited to price only non-dividend European firms. Along with Myron Scholes and Fischer Black, Robert Merton was awarded the Nobel Prize in 1997 for ground breaking quantitative analysis.

Variable	Description
S	Stock Price
X	Exercise Price
T	Time of option contract
R	Risk free rate
D	Dividend yield
ó	Volatility of stock price
Ln	Natural Logarithm
N(d)	Cumulative distribution function
E	Exponential function

Price of Call Option = $\frac{S}{e^{dt}}$ $N(d_1) - \frac{X}{e^{rt}} N(d_2)$

Price of Put Option = $\frac{X}{e^{rt}} N(-d_2) - \frac{S}{e^{dt}} N(-d_1)$

$$d_1 = \frac{ln\left(\frac{S}{X}\right) + \left(r - d + \frac{\sigma^2}{2}\right)(t)}{\sigma\sqrt{t}}$$

$$d_2 = \frac{ln\left(\frac{S}{X}\right) + \left(r - d - \frac{\sigma^2}{2}\right)(t)}{\sigma\sqrt{t}}$$

METHODOLOGY

This paper aims to value option premiums by using the above models not for the sole purpose of valuation but if the models can be utilized to identify potential real time options. Valuation of any financial product is performed to identify over-priced and under-priced financial securities. Under-priced securities generate buy signals whereas over-priced securities provide short selling opportunities or and indication to stay away from the market. Investors prone to firm fundamentals often rely on valuation models to make an investment decision. Ten of the most well known large cap U.S firms from various sectors have been selected for the valuation. These include: International Business Machine, Wal-Mart, Microsoft, Pfizer, Coca Cola, McDonalds, Boeing, JP Morgan, Google and Caterpillar. Data for the variables were gathered from secondary sources extracted from the National Association of Security Dealers Automated Quotations online website. The volatility has been derived from the monthly closing share prices for the past twelve months. The discount rate of U.S Treasury Bills has been used for the interest rate. The data's were then integrated into the equations to arrive at the numeric's to be analyzed with the actual market prices for both calls and puts.

EMPIRICAL FINDINGS

Table I: Value of Calls and Puts (Value in $ per share)

Market Price	Strike Price	Firm	B-S-M Value		Binomial Value		Actual Market Price			Expiration
			Call	Put	Call	Put	Call	Put	Date	
61.71	90	IBM	0	29.06	0	30.066	0.03	31.6	29-Sep-11	22-Jan-11
53.36	55	WMT	0.00509	2.123	0	1.64	0.83	1.38	1-Oct-10	18-Dec-10
24.38	25	MSFT	0.00996	0.8907	0	0.62	1.56	2.5	1-Oct-10	16-Apr-10
17.17	17	PFE	0.01329	0.1898	0.289	0	1.22	1.31	1-Oct-10	19-Mar-11
59.12	52.5	KO	5.78	0	6.957	0	7.3	1.89	1-Oct-10	5-May-11
74.92	70	MCD	3.89	0.155	5.369	0	5.6	1.03	1-Oct-10	18-Dec-10
66.83	62.5	BA	3.856	0.236	4.3707	0	7.85	3.85	1-Oct-10	19-Feb-11
38.81	38	JPM	0.84167	0.00268	0.872	0	2.22	1.44	1-Oct-10	20-Nov-10
525.62	580	GOOG	155.965	196.16	193.22	233.41	48.2	111.1	1-Oct-10	21-Jan-12
78.22	75	CAT	3.11496	0.19743	2.9223	0	3.5	0.83	4 Oct-10	16-Oct-10

The above table displays the empirics extracted from the equations. The first two columns provide the exercise prices and the market prices of the ten firms. As the market prices are subject to change in every second from 9:30am till 4:00pm Monday to Friday, the date of the market prices is provided in the second last column, which is also the same price used in the valuation models. The fourth and fifth column displays the values of option premiums obtained in U.S dollars per share. In total, forty valuations have been performed. The findings provide mixed views. For some firms, the model successfully values the premium which is very

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close to the actual whereas in others, the values provide doubtful investment signals. Of all ten firms, IBM is the only firm where both Binomial and Black-Scholes-Merton model gave accurate prices for the puts. Caterpillar and Coca Cola's Black Scholes call provided close results along with Wal-Mart's put. Out of forty, only five provided results that were close to the actual, the rest thirty five were mixed with undervalued and overvalued pricing. In several of the binomial valuations, puts of six companies gave zero valuation as the exercise price was greater than the market share price. However, those particular puts had significant market values. Three of the Binomial calls also provided same results as the strike price was greater than the market share price while the market had a price for the call options. The Black-Scholes-Merton provided results better than the Binomial model but still inaccurate as the valuation prices were either highly overvalued or undervalued.

CONCLUSION

Financial valuation models tend to indentify potential securities that can provide profitable returns. This paper utilized two of the most well known option pricing models. Out of forty valuations, correct pricing accounted to a mere 12.5%. Theoretically, options that have no intrinsic values should not have any market values. In the case of a call option, a return is generated when the market price is significantly greater than the exercise price. On the opposite end, when the exercise price is greater than the market price, the option is considered to be out of the money thus implying theoretically, that the option should have no value. For a put option, a return can be generated if the exercise price is significantly greater than the market price. A put option theoretically is posed to have no value if the market price is greater than the strike price. Both the models were unable to provide any value for the options that were out of the money, on the contrary the market had a price for those options. The concluding fact is that both the models cannot be utilized in options that have no intrinsic value. Options that have intrinsic values are more favorable for the models, although the values differed from the market price. Since the models provided inaccurate results for the options that had zero intrinsic value, it can be assumed that the values obtained for the options with some intrinsic value may not be accurate as well. A similar study conducted by Geske and Roll (1984) on valuing American options by the Black-Scholes model provided conflicting results. In 1998, Long Term Capital Management, a hedge fund in U.S lost over $4.5 billion applying mathematical models including the Blacks-Scholes in indentifying potential derivative investment opportunities. The hedge fund had Myron Scholes and Robert Merton as board members along with other prominent figures from the investment arena. The fact is, although the models are undoubtedly empirically brilliant, but its application in identifying potential investments in stock options may not provide the best of results.

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